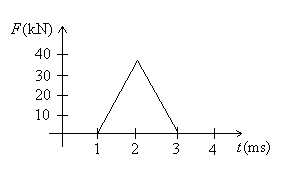
**Homework 8 Solutions due never**

**Problem 1**. A pitcher throws a baseball toward the batter in the negative x direction with a speed of 40 m/s. The batter swings and makes contact with ball, which experiences a force that varies over time according to the graph shown below. What is the final velocity of the ball? Assume that the baseball has a mass of 0.25 kg, and assume everything takes place along the x-direction. And take note of the units on the axes: kN = 1000N, ms = 0.001s.

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Use the J-p equation. We’ll note that the impulse is given by the area of the F vs. t curve, which is Jbat = (1/2)(0.002s)(40 000N) = 40N·s. And the initial velocity is -40m/s. Therefore the final velocity is:



**Problem 2.** A rocket propels itself forward by using chemical reactions to eject superhot gas out the back of the engine. Suppose the engine is burning fuel at the rate of 1200kg/s, and ejecting it out the back at a speed v = 800m/s (this is the speed of the fuel, *not* the rocket). Let the rocket itself have a mass m = 20 000kg.



(a) What is the impulse the engine exerts on the gas every second?

This would be:



(b) What impulse does the gas exert on the engine (and rocket, by extension), therefore, every second?

This is the same, J = 960 000 N∙s

(c) What is the force the gas exerts on the rocket? This is called the thrust.

Force is F = J/Δt = 960 000 N∙s/1s = 960 000 N.

(d) What is the acceleration of the rocket? Don’t neglect gravity here.

Acceleration is:

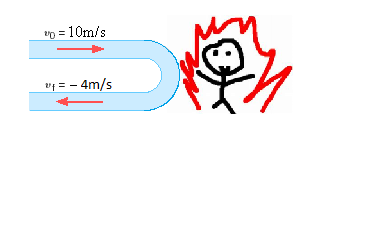


(e) A typical ‘burn’ lasts a couple minutes. How fast would the rocket be going after 3 minutes, assuming it started from rest on the launchpad? Note that it would actually be going faster than this, since as it ejects fuel, it gets lighter, and therefore accelerates at an increasing rate.

So we have, according to usual kinematics,



**Problem 3.** You find yourself on fire, so fire, and a fireman sprays you down with a hose, striking you with water at the rate of 20kg/s. The stream of water hits you horizontally with a speed of 10m/s, and reflects backwards with a speed of 4m/s.



(a) What is the impulse you’re exerting on the water every second?

From the J-P equation:



(b) What is the force you’re exerting on the water every second?

This is:



(c) What is the force the water is exerting on you every second?

This is equal and opposite to your force. So,



**Problem 4.** Two iceskaters on a frozen pond and standing still, side by side, and then push off each other. Suppose iceskater L (on the left) has a mass m1 = 80kg, and iceskater R (on the right) has a mass m2 = 60kg. Suppose there is no friction between their skates and ice.



(a) Is the force R exerts on L less than, equal to, or greater than the force L exerts? Or is it impossible to say? Does your answer change if there is friction between skates and ice?

Equal to, by N3L. Doesn’t change with friction.

(b) Is the impulse R exerts on L less than, equal to, or greater than the impulse L exerts. Or is it impossible to say? Does your answer change if there is friction between skates and ice?

Equal to, by N3L, since the times they exert their forces are identical. Doesn’t change with friction.

(c) Is the change in momentum R experiences less than, equal to, or greater than the change in momentum L experiences? Or is it impossible to say? Does your answer change if there is friction between skates and ice?

Equal to, since impulse equals change in momentum. With friction, it’s impossible to say, because friction will exert its own impulse.

(d) Is R’s final velocity less than, equal to, or greater than L’s final velocity. Or is it impossible to say? Does your answer change if there is friction between skates and ice?

R’s final velocity is greater, since R has a smaller mass. With friction, it’s impossible to say, because friction will exert its own impulse.

(e) Is theit total momentum conserved? Does your answer change if there is friction between skates and ice?

Yes, since there is no external force acting on them in the x-direction to change any of their momentum.

But with friction it’s impossible to say.

**Problem 5.** You’re standing on a paddle board. Let’s say its mass is mboard = 20kg, and its length is 3m. And suppose your mass is 80kg, and you’re standing on the left edge at x = 0 (unlike the guy in the picture who is standing in the middle-ish).



(a) Now say you start running down the length of the board. You’ll be moving right, and the board will be moving left. Which of the following conditions is necessary to assume that total momentum is conserved in the x-direction:

1. Friction between your feet and the board is negligible.

2. Friction between the board and the water is negligible.

Friction between your feet and the board is an internal force, and so wouldn’t affect the total momentum in the x-direction. But friction between the board and the water is an external force, and would affect the total momentum. Only if this is negligible will momentum be conserved.

(b) Suppose you start running down the board with a speed of 2m/s. How fast is the board moving to the left?

So we’ll use momentum conservation of course!



(c) If you multiply both sides of your momentum conservation equation, above, by time t, then you’d get the following equation:



Use this last equation to determine how far you’ve gone to the right, by the time you reach the end of the board. And how far the board has gone to the left. You may take the initial positions of the board, and you, to be x = 0.

So we have:



And then we also have the following relationship, that when you’re at the end of the board, your position will be 3m further to the right than its position:



Filling xyou into the first equation we have:



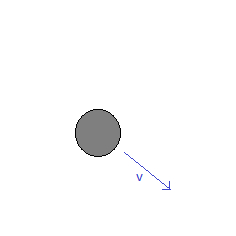
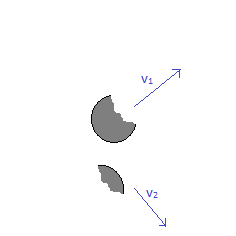
And then



(d) Eventually both you and the board will come to rest after hitting the water. What will be your total momentum then, and where did it go?

Your total momentum will be zero. And it was absorbed by the water, via the resistive force it exerts on you, and the board.

**Problem 6.** A 20kg cannonball, traveling in air 30° below the horizontal, at v = 150m/s, prematurely explodes, sending a 15kg fragment traveling 30° above the horizontal with a speed of v1 = 100m/s. You may ignore air resistance.

(a) What was the initial momentum in the x-direction? What was the momentum in the x-direction right after the explosion? What was the momentum in the x-direction 5s after the explosion?

Well,



This will be the same afterwards, and 5s later, as there is no external force in the x-direction to change it.

(b) What was the initial momentum in the y-direction? What was the momentum in the y-direction right after the explosion? What was the momentum in the y-direction 5s after the explosion?

And this is,



This will be the same, right after the explosion. But not so 5s afterwards. Gravity exerts a downward force, and so the total momentum in the y-direction will be:



(c) What was the speed and direction of the 5kg fragment right after the explosion?

We can use momentum conservation. So:



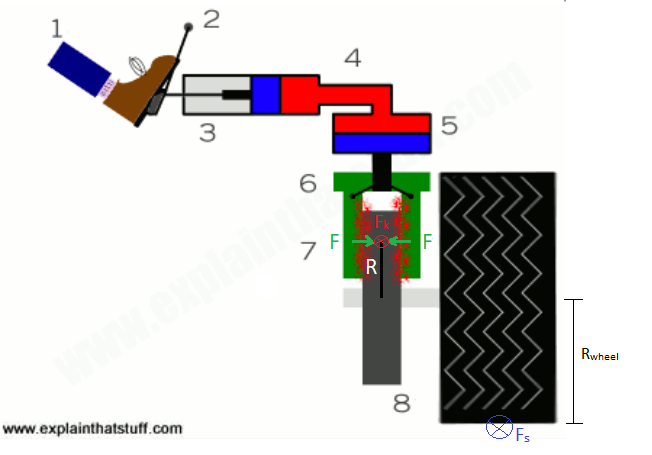
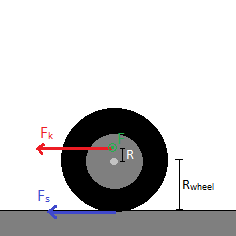
And,



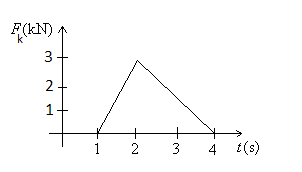
So the overall speed and direction of the fragment was:



**Problem 7.** You see a talking doll on the road in front of you. You should just speed up and smash it into a million pieces. But instead, like all people in horror movies, you step on your brakes, slow down and get out of your car. So about the brakes…when you exert a force on the brakes, the brake fluid (red stuff) magnifies your force via Pascal’s principle (PHY 122 stuff), and makes the brake pads (green stuff) clamp down onto the brake disk (the dark grey cylinder to the left of the tire) with a force we’ll call F. This will cause a kinetic friction force Fk = μkF (and there’s one on each side of the disk) to slow down the disk, and consequently the wheel, as the disk and wheel are connected to each other via that light grey axle. Additionally, the static friction force will act on the bottom of the wheel, opposing what the brake pads are attempting to do to the wheel. Two perspectives are shown below:

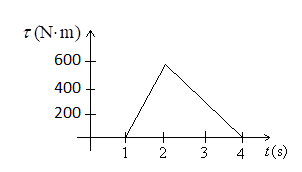
 

Let’s say the wheel+disk’s moment of inertia is I = 0.9kg∙m2, and that it is initially turning clockwise at ω = -67 rad/s. Let the radius of the wheel be Rwheel = 30cm, and the distance between the wheel’s axle and brake pads be R = 20cm. We’ll posit that the static friction force is given by Fs = 1960N, and that the kinetic friction force is given by the graph below, as you press and release the brakes.

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(a) What is the angular impulse the brake pads exert on the disk, and equivalently, on the wheel? Remember there’s two pads.

The angular impulse is the area under the τ vs. t curve. Since R = 0.20m, the torque is τ = RFksin(90°) = (0.20)Fk. And so for instance, the top torque would be τ = (0.20)(3000) = 600N∙m. And so the graph would look like this:

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And the area is KFk = (1/2)(3s)(600N∙m) = 900N∙m∙s. And so both pads together would exert an angular impulse KFk = 1800N∙m∙s.

(b) What is the angular impulse exerted by the static friction force?

Fs exerts a constant torque τ = -FsRwheelsin(90°) = -(1960)(0.30) = -588N∙m. And so the impulse is KFs = τΔt = (-588N∙m)(3s) = -1764N∙m∙s.

(c) What is the angular speed of the tire?

We have to use the angular impulse-momentum equation:



**Problem 8.** Suppose you (m = 60kg) are on the rim of a merry-go-round (shaped like a hoop with M = 100kg, R = 1.5m) making one revolution every second. If you walk to the center of the merry-go-round, what will be the new rate of rotation? Assume that the axle upon which the merry-go-round turns is frictionless so that it will not exert a torque.

Since there will be no net torque acting on our system of objects then, we’ll have,



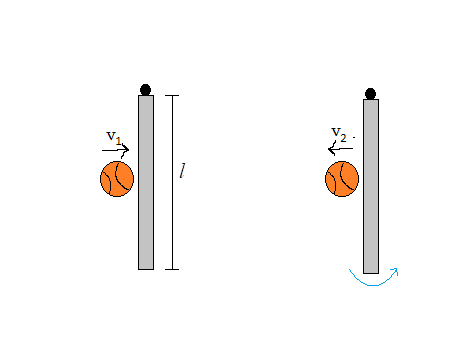
The initial angular momentum is that of the merry-go-round and the person standing on the edge. The final is that of the merry-go-round alone, since the person will have moved to the center and their radius w/r to the axis of rotation will be zero. So we have,



To find the frequency of rotation, we divide by 2π, which is the number of radians per revolution.



**Problem 9.** Too lazy to get up and open your door (Mdoor = 40kg, ℓ = 1.3m) when your friend comes by, you instead throw a basketball (mball = 1.2kg) at its midpoint, with a speed v = 16m/s. And say it rebounds with half that speed. How long will it take the door to open 90°? You can take the hinge to be frictionless. Note the door isn’t rotating about its center (so parallel axis theorem)…and for the ball’s moment of inertia, you can just use the mh2 term in the parallel axis theorem. Finally, to get the ball’s angular velocity about the hinge at the moment of collision, recall v = ωr.



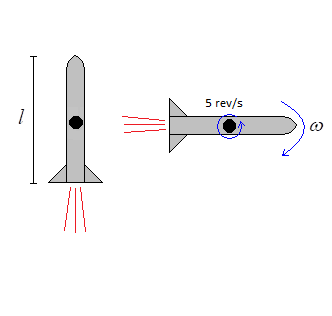
Have conservation of angular momentum again. Note the moment of inertia of the door must be obtained with the parallel axis theorem, since it’s not rotating about its midpoint. So Idoor = Idoor,cm + Mh2 = (1/12)Mℓ2 + M(ℓ/2)2 = (1/3)Mℓ2 = (1/3)(40)(1.3)2 = 22.5kg∙m2. Also, the moment of inertia of the basketball about the axis of rotation (hinge) will be Iball = mh2 = m(ℓ/2)2 = (1.2kg)(1.3m/2)2 = 0.51kg∙m2. And its initial angular velocity will be ωball,i = vi/r = vi/(ℓ/2) = 16/(1.3/2) = 24.6 rad/s. Its final angular velocity will be ωball,f = -12.3rad/s. So then,



And so the time it would take to turn 90 degrees = π/2 rad is,



**Problem 10.** Let’s say you have a rocket ship (M = 10 000kg, ℓ = 25m) that is traveling ‘straight’. But we need to maneuver it to the right. How can this be done if the engine nozzle can’t be tilted? A simple way to do this is with a gyro. Suppose it is shaped like a disk (m = 100kg, R = 20cm), and an internal motor begins rotating it counter-clockwise at the rate of 5 rev/s. Then by angular momentum conservation, the rocket must start rotating clockwise at some rate ω.



(a) Which of the following is required for angular momentum to be conserved?

1. Friction in the axle about which the gyro turns is negligible.

2. Friction between the rocket and ‘space’ is negligible.

3. The engine must be turned off.

Friction is an internal force, in this case, much like it was in the paddle-board example. So it needn’t be negligible. Friction between the rocket and ‘space’ must be negligible since that would be an external friction force. Fortunately this *is* negligible since space is mostly empty. The engine needn’t be turned off though. The thrust it exerts points directly through the axis of rotation, and so even though it would count as an external force of sorts, the torque it exerts would be zero, and so it’s angular impulse would be zero, and so it wouldn’t change the angular momentum.

(b) Given the rate of rotation of the gyro, how fast will the rocket be rotating?

Since angular momentum is conserved, we have:



(c) Just like in the paddleboard example, we may multiply our conservation of momentum equation to get the following relationship between the angular positions of the gyro and rocket:



Using this last equation, determine how many revolutions the gyro must turn through, before rocket has turned 90 degrees. And how long would this take?

So,



Δt is given by:

